Written Exam at the Department of Economics Summer 2018

Micro III

Final Exam

June 12, 2018

(2-hour closed book exam)

Answers only in English.

This exam question consists of 3 pages in total (including the current page).

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Be careful not to cheat at exams!

- You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

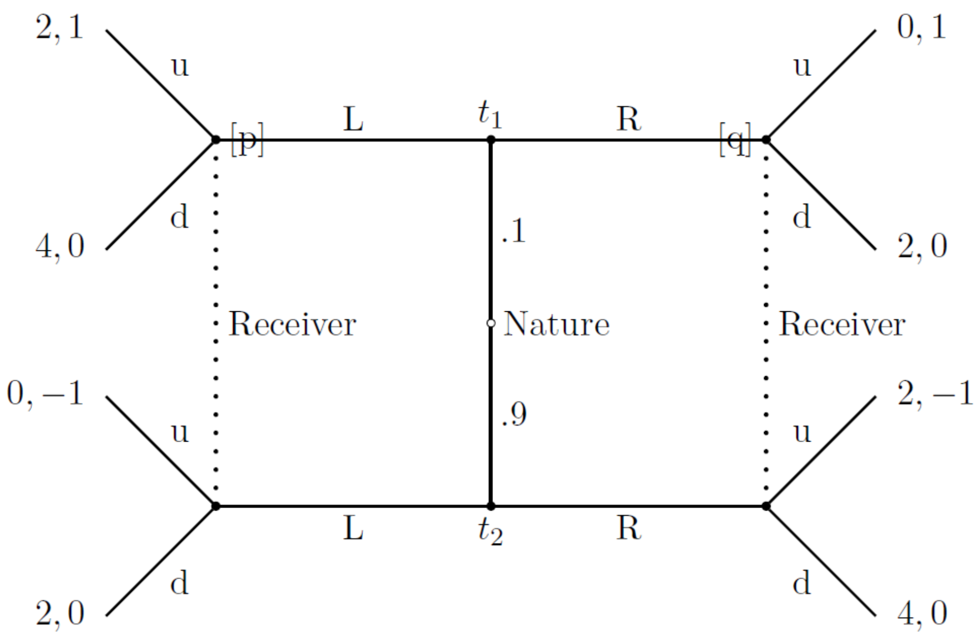
1. Consider a first-price sealed bid auction with $N \geq 2$ bidders, who have valuations $v_1, v_2, \ldots, v_N$. Each value v_i, $i \in \{1, 2, \ldots N\}$ is independently drawn from a uniform distribution on $[0, 1]$. Values are **private**.

   (a) Show that there is a symmetric Bayesian Nash equilibrium in linear strategies, $b(v_i) = cv_i$, by explicitly solving for the value of c.

   (b) Compare your answer in part (a) to the answer you would obtain in the special case with two bidders, $N = 2$. Briefly comment on any similarities and differences (2-3 sentences).

   (c) What would happen to bidding behavior in this auction if the number of bidders became very large (say if N increased without bound)? Intuitively describe, in words, why this is the case, and what you expect would happen to the expected revenue of the seller (2-4 sentences). *Please attempt this question even if you did not successfully complete the earlier parts.*

2. Now consider the following game G:



Note that in this game, the prior probability that the sender is of type 1 is equal to 0.1.

   (a) Briefly explain whether G is a static or a dynamic game (1 sentence), and whether or not G is a cheap talk game (1 sentence).

   (b) Find a separating equilibrium in G, and find a pooling equilibrium where both sender types play L.

   (c) Check whether the equilibria you found in part (b) satisfy Signaling Requirement 6 ('*equilibrium domination*').

   (d) Describe a real-world strategic situation that could correspond to G, and explain why this is the case (3-4 sentences). What (if anything) do your answers to parts (b) and (c) suggest about the behavior we are likely to see in this real-world situation? (2-3 sentences)

3. Consider a static game F where two firms produce a homogeneous good and compete in quantities. Firm 1 and Firm 2 both produce at zero cost. Let q_i denote the quantity produced by Firm $i \in \{1, 2\}$. Given q_1 and q_2, the market price is $p = 4 - q_1 - q_2$. Both firms choose quantities simultaneously, and maximize profits.

   (a) Solve for the Nash equilibrium of this game. What profits do Firm 1 and Firm 2 earn in equilibrium? What profits would Firm 1 and Firm 2 earn if they instead each produced half of the monopoly quantity (i.e. half of the quantity that maximizes total industry profits)?

   Now consider a dynamic game, with infinite time horizon, where Firm 1 and Firm 2 play the stage game F in periods $t = 1, 2, 3, \ldots$. You can assume that both firms discount future payoffs with factor $\delta \in (0, 1)$.

   (b) For what values of δ can you find a subgame perfect Nash equilibrium where, on the equilibrium path, each firm produces half of the monopoly quantity in each period? Explicitly solve for this equilibrium, making sure to state each firm's strategy, and show that neither firm has an incentive to deviate.

   Now assume that there is an anti-trust authority in this market which acts in the following way: in any period t where the total quantity produced, $q_1 + q_2$, differs from the monopoly quantity, the anti-trust authority does nothing; in any period t where the total quantity produced $q_1 + q_2$ is equal to the monopoly quantity, there is a probability $p \in [0, 1)$ that the anti-trust authority detects collusion. If collusion is detected, then the anti-trust authority cannot impose a fine, but instead forces both firms to leave the market at the end of period t (so that both firms then earn zero profits in period $t + 1, t + 2, \ldots$). Notice that in the special case with $p = 0$, collusion is never detected, so the anti-trust authority plays no role, and is effectively absent from the market.

   (c) Consider the dynamic game described above where the anti-trust authority can detect collusion with probability $p \in [0, 1)$. Write down an inequality, involving p and δ, which implicitly defines the values of δ for which neither firm has an incentive to deviate from the equilibrium you found in part (b) (you do **not** need to explicitly solve this inequality to isolate δ).

   (d) Using your answers in parts (b) and (c), comment on how the presence of the anti-trust authority may (or may not) impact firm behavior in this market, compared to a setting where the anti-trust authority was absent. Does your answer depend on the value of p, and on whether the value of the discount factor is low, intermediate, or high (3-4 sentences)? Briefly give the intuition behind your answer (2-3 sentences). *Please attempt this question even if you did not successfully complete the earlier parts.*